Simulink Model of an Induction Machine

This document describes the defining equations for a dynamic induction machine model to be implemented in Simulink. There are a number of research publications on this topic however they often lack key pieces of information which the author either decided were so self-evident they need not be included or just neglected due to their familiarity with the subject matter. Something that may be about to happen again.

In terms of research into the design and control of electrical machines the process described here is "classical" (i.e. more than a few years old). In practice a number of companies known to the author use a process similar to this for several types of machine drive but with additions to accommodate effects particular to their application. For example in high speed work the frictional heating of the rotor due to wind-age and aerodynamic losses may be included as these are often a function of speed. Saturation of the machine iron may be included by parameterising the leakage and mutual inductances as a function of flux or current. Copper and Eddy current losses are relatively easily added and thereafter the thermal effects that these create may be included as well. The final form of the simulation and its scope is limited only by technical necessity, the human resource available to perform the development work and the financial cost the developer is prepared to incur to possess the simulation. The main cost of developing a simulation for a machine is the validation stage(s) which require physical space, test equipment and human resources to gather the validation data. By comparison the programming of the model including the HR costs of the programming and the cost of the software is often a minor component of a project.

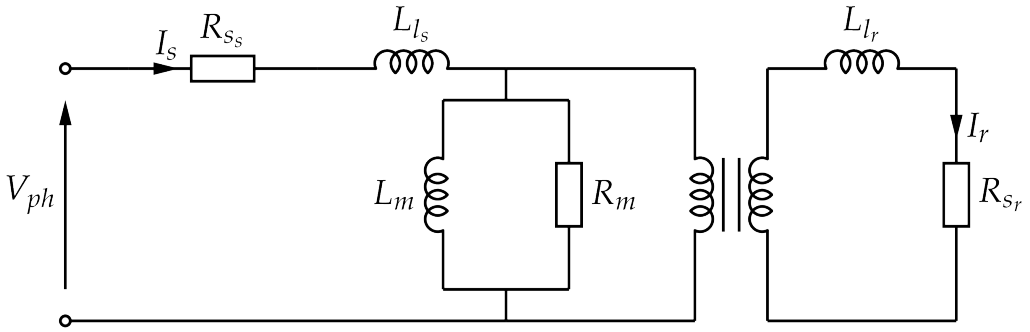


Figure 1: Per phase equivalent circuit of an induction machine.

The three phase induction machine per phase equivalent model which is commonly taught to undergraduates is shown in fig. 1. A discursive introduction to machines and control may be found in [1]. Some common questions that undergraduates might face involve referring the rotor to the stator side and deriving the torque equation, the pull out torque, the starting torque and sketching a

graph of the torque/power speed curve. This can be done without the necessity of teaching any field oriented control material. The difficulty with this approach is that it works well for setting exams but not so well for practical control of machines or for doing simulations of the machines as if they were systems. It is not common to perform control calculations in the *abc* reference frame.

In the case where one is performing magneto-static finite element simulations to interrogate the machine design, the control aspects of the system are often not of great concern and working in *abc* is usually acceptable. it is common to inject sine wave currents into the phase windings to develop the flux density plots and thereafter derive the torque speed and efficiency and ultimately the torque per amp or per unit volume etc. for a given rotor and stator geometry, magnetic material and winding configuration etc.

Relatively recently it has become popular to link the control and power electronics simulations with FE simulation 'in the loop' i.e. one clock tick of the control and power electronics simulation in Simulink gives rise to a set of voltages on the stator windings which are passed to the FE program. The FE program performs one clock tick (often of the same length or a number of ticks that make up the same length as the control and power electronics simulation) and solves the magneto-static equations to provide the current that will flow in the machine windings as a result of the applied voltage. These currents are used by the control and power electronics simulations to calculate, for example, the $I\,R$ drops across the power switches (among many other things) and this gives rise to the applied voltage on the windings in the next time-step. This coupled simulation capability exists in a number of proprietary packages including Flux, Ansys and COMSOL. Although, COMSOL's mechanism of interacting with MATLAB is not as easily amenable to the per clock tick coupling described here as for example Flux's method, nevertheless, it is possible. Others such as Vector Field and JMAG have something similar. In principle such a coupling should be possible between OpenModelica and Elmer or OpenFOAM, but I am unaware of any existing implementation.

From a control systems perspective it is preferable to mode most machines as if they are DC machines. When there are steady state quantities representing energy transfer in the DC machine the currents and voltages that represent that energy on the electrical side are constant i.e. not time varying (like sine waves). This is desirable because it makes the calculations easier for us to interpret, and the control optimisation less challenging. More importantly it is potentially less computationally demanding as well, remembering that in an industrial drive the embedded system should be as inexpensive as possible but still able to complete the task effectively.

To facilitate this approach to modelling the AC machines as DC machines the

work of Clark and Park is used either in the Clarke Park transform or the *dq* transform. This transform converts the three phase currents of the stationary *abc* frame to the stationary α, β reference frame and then to the rotating d-q reference frame. If a refresher is needed then there is an excellent video at https://www.youtube.com/watch?v=vdeVVTltr1M (not one of mine and I'm not affiliated with the author). The transformation may be performed easily in Simulink using a built in block. It is not necessary to describe the transform mathematics here, it may be found in textbooks including [2–4]. A discussion of PWM techniques can be found in [5] and [6].

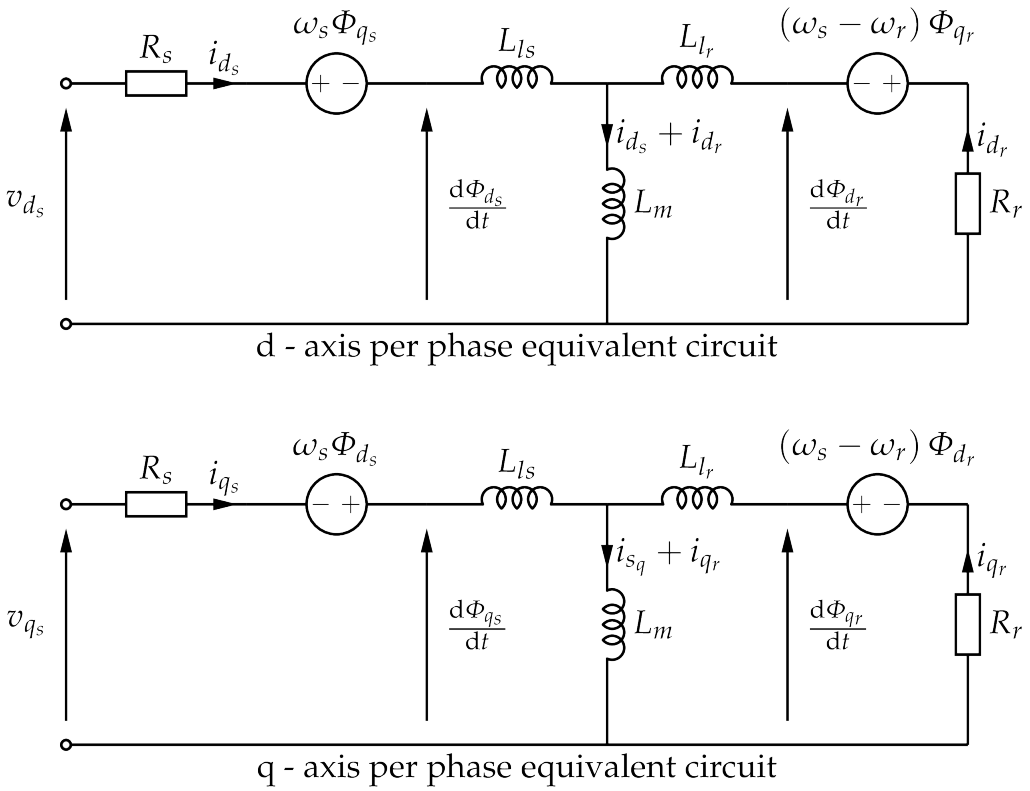


Figure 2: d-q axis decomposition of an induction machine.

The d-q transformed equivalent circuit is shown in Fig. 2 where the nomenclature is given in table 1 and the value column provides representative parameter values for a 1.1 kW machine used in a light industrial machine tool.

Stator series resistance The stator series resistance is a representation of the finite DC resistance of the stator winding.

Stator leakage inductance The stator leakage inductance represents the magnetic flux surrounding the stator windings (coils), due to the current passing

Parameter	Description	Value	Units
R_s	Stator winding series resistance	6	Ω
L_{ls}	Stator winding leakage inductance.	30	mH
L_m	Mutual inductance coupling stator and rotor	500	mH
L_{lr}	Rotor winding leakage inductance	30	mH
R_r	Rotor winding resistance	6	Ω
Power	Rated average power of the machine	1.1	kW
Voltage	Rated phase to phase voltage of the machine	415	V
Current	Rated phase current of the machine	2.77	A
Frequency	Rated frequency	50	Hz
Speed	Rated speed	1415	rpm
Poles	4 poles (2 pole pairs)		
J	Rotor inertia	0.02	$\mathrm{kg\,m^2}$
D	Total Friction/Drag	0.003	$\frac{\mathrm{Nm}}{\mathrm{rad\,s^{-1}}}$

Table 1: Nomenclature and representative values for the d-q decomposition of a 1.1 kW induction machine.

through the stator windings, which does not cut the rotor bars/windings and therefore does not have the opportunity to induce a current in the rotor. This inductance represents the portion of the stator flux which does not manage to couple the stator to the rotor. It is sometimes said to be due to the 'end winding(s)'. Winding schemes with shorter end windings tend to suffer lower leakage inductances. The advantages and disadvantages of particular winding schemes is an active area of machine design research. For example, in SR machines [7]. However, in commercial applications the cost of applying the winding to a machine is usually the most important factor unless there are special design criteria that must be met. Accepting a slightly longer end winding while having an easy i.e. automated mechanism of applying the winding to the machine is a common industrial trade-off.

Rotor series resistance The rotor series resistance is a representation of the finite DC resistance of the rotor winding or bars. The rotor is often a squirrel cage design having bars rather than windings but may be wound in some circumstances.

Rotor leakage inductance The rotor leakage inductance represents the magnetic flux surrounding the rotor bars or windings, due to the current induced in the rotor by the stator magnetic field, which does not cut the stator coils. This is effectively a representation of the part of the rotor flux which does not link the rotor to the stator.

Stator back EMF The d axis stator back EMF is proportional to the synchronous speed and the q axis flux. The current gives rise to a magnetic flux and the rate of change of that magnetic flux gives rise to a voltage across the winding in which the current flows the back EMF will act to oppose the change in current (Lenz's law). If the current is sinusoidal the voltage that develops will be co-sinusoidal (derivative of sine is cosine) cosine may be thought of as a sine wave with a phase shift of $\pi/4$. Since the d and q axes are perpendicular to each-other (shifted $\pi/4$ radians) the EMF felt on the d axis is due to the q axis flux and similarly the EMF felt on the q axis is proportional to the d axis flux.

Rotor back EMF The d axis rotor back EMF is proportional to the difference between the synchronous speed and the rotor speed (i.e. the "slip") and the rotor flux generated by the q axis. The difference in the synchronous and rotor speed is the rate at which the stator field appears to be moving past us if we're sitting on the rotor bars looking radially outwards.

Figure 2 also notes the voltage across the stator windings ($L_{ls} + L_m$) and rotor windings ($L_{lr} + L_m$) is given by the rate of change of flux cutting those windings.

Kirchhoff's laws may be used to develop a set of loop equations for the stator and rotor. Later these equations will be combined with flux equations to form a self consistent set of equations that Simulink (or Modelica) can solve. The mechanical system will also be added to the simulation. Starting with the d axis stator loop and recalling that the voltage across an inductor is given by the product of the rate of change of current and the inductance of the coil ($v = L\,\mathrm{d}i/\mathrm{d}t$),

$$v_{ds} - i_{ds}\,R_s + \omega_s\,\Phi_{qs} - L_{ls}\,\frac{\mathrm{d}\,i_{ds}}{\mathrm{d}t} - L_m\,\frac{\mathrm{d}}{\mathrm{d}t}\,(i_{ds} + i_{dr}) = 0 \tag{1}$$

The relationship between flux and current (flux is the product of inductance and current) can be used to simplify (1), although this can be obtained by using the voltages marked by the rate of change of flux in fig. 2 as well

$$v_{ds} - i_{ds}\,R_s + \omega_s\,\Phi_{qs} - \frac{\mathrm{d}\,\Phi_{ds}}{\mathrm{d}t} = 0 \tag{2}$$

transposing (2) to make $\frac{\mathrm{d}\,\Phi_{ds}}{\mathrm{d}t}$ the subject,

$$\frac{\mathrm{d}\,\Phi_{ds}}{\mathrm{d}t} = v_{ds} - i_{ds}\,R_s - \omega_s\,\Phi_{qs} \tag{3}$$

The expressions involving differential equations are usually cast in the form of derivative = everything else, because that is the way the Simulink blocks will be arranged. After the equation is formed the 1/s block is used to integrate the differential equation. The differential equations are coupled and the solver

that Simulink calls will solve them simultaneously. Well, presuming the solution converges. If the simulation doesn't converge it's quite likely that one of the equations is either derived incorrectly or has not been transferred into Simulink correctly. For the q axis,

$$\frac{\mathrm{d}\,\Phi_{qs}}{\mathrm{d}t} = v_{qs} - i_{qs}\,R_s + \omega_s\,\Phi_{ds} \tag{4}$$

The rotor loops can be similarly treated, starting with the d axis,

$$\frac{\mathrm{d}\,\Phi_{dr}}{\mathrm{d}t} = -i_{dr}\,R_r - (\omega_s - \omega_r)\;\Phi_{qr} \tag{5}$$

and for the q axis,

$$\frac{\mathrm{d}\,\Phi_{qr}}{\mathrm{d}t} = -i_{qr}\,R_r + (\omega_s - \omega_r)\;\Phi_{dr} \tag{6}$$

Four expressions link the d and q axis rotor and stator flux starting with the q axis stator flux,

$$\Phi_{sq} = (i_{qs} + i_{qr})\;L_m + i_{qs}\,L_{ls} \tag{7}$$

for the d axis stator flux,

$$\Phi_{sd} = (i_{ds} + i_{dr})\;L_m + i_{ds}\,L_{ls} \tag{8}$$

for the q axis rotor flux,

$$\Phi_{rq} = (i_{qs} + i_{qr})\;L_m + i_{qr}\,L_{lr} \tag{9}$$

for the d axis rotor flux,

$$\Phi_{rd} = (i_{ds} + i_{dr})\;L_m + i_{dr}\,L_{lr} \tag{10}$$

In this case the flux equations may be written in terms of the d and q axis currents. This is necessary because the simulation will proceed by a voltage being impressed on the stator winding. This voltage will be used to calculate the rate of change of flux, which in turn will be integrated to obtain that flux. The flux will then be used to calculate the current. The current will be fed back into a controlled current source which exists between (i.e. in parallel with) the terminals across which the voltage was originally impressed. It's possible to formulate another approach in which a current is impressed and the resulting EMF on the stator is calculated (along with all the mechanical outputs) however this requires the system of equations to be cast in terms of calculating derivatives. Taking derivatives in numerical sampled data systems is potentially risky because it is a noisy process which can promote instability (i.e. lack of convergence). Integration or numerical quadrature is by comparison a low noise and generally stable approach. There are some circuit reasons to use an integration approach as well.

When the simulation is set up with some power electronics the machine model will take the calculated current independent of the other effects in the system. If a back EMF approach was used the converter supply voltage would have the machine terminal voltage subtracted from it and the machine current would be whatever flowed through the series resistance of the power semiconductor device. If the power device model has no series resistance and the link voltage was a perfect voltage source there will be an over-constrained matrix in the simulation as two voltage sources will have been connected in parallel which is undesirable. Keeping in mind all the foregoing, if the converter design under consideration is a current source converter or an impedance source converter it's not impossible, it may even be desirable, to cast the machine model as being a controlled voltage device. However, in voltage source inverter applications, which are much more common, presenting the machine model to the power electronics model as a controlled current device is certainly preferable. This should be clearer when considered in association with the video demonstrating the Simulink code (see https://youtu.be/wM72tarF_to).

The q axis stator flux expression (7) becomes,

$$i_{qs} = \frac{\Phi_{qs} - L_m\, i_{qr}}{L_m + L_{ls}} \tag{11}$$

Equation (8) is transposed to provide the d axis stator current,

$$i_{ds} = \frac{\Phi_{ds} - L_m\, i_{dr}}{L_m + L_{ls}} \tag{12}$$

Two more expressions are require and can be derived trivially from (9) and (10) to provide the d and q axis rotor currents in terms of the d and q axis rotor flux. Looking at (11) and (12) it is clear that the solution of these two equations and the two which are not written out must be simultaneous as they all depend on each-other. This is similar to the differential equations for flux, they are interdependent and the solver must solve them all simultaneously.

The electro-mechanical torque can be expressed as,

$$T_e = \frac{3}{2}\frac{P}{2}\left(i_{qs}\left(\left(i_{ds} + i_{dr}\right)\, L_m\right) + i_{ds}\left(\left(i_{qs} + i_{qr}\right)\, L_m\right)\right) \tag{13}$$

The electro-mechanical torque could be expressed in terms of flux more simply but the Simulink model that this document relates to uses the form presented here. Since the integral of the time derivative of flux is calculated prior to the current the Simulink blocks flow in a more aesthetic way if the current is used to develop the torque. The rate of change of speed is given by,

$$\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = \frac{T_e - D\,\omega_r - T_L}{J} \tag{14}$$

where ω_r is the rotor angular velocity in $\mathrm{rad\,s^{-1}}$, T_e is the electro-mechanical torque in Nm, D is the frictional drag in $\mathrm{Nm/(rad\ s^{-1})}$, T_L is the load torque, Nm (which when positive opposes the electro-mechanical torque) and J is the total mechanical inertia of the rotating parts in $\mathrm{kg\,m^2}$. The position of the rotor, in radians, is the integral of the rotor angular velocity,

$$\theta = \int \omega_r \, \mathrm{d}t. \tag{15}$$

The implementation in Simulink is presented in a video at https://youtu.be/wM72tarF_to

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